LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18UMT1MC01– ALGEBRA AND CALCULUS - I

Date: 24-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions: **(10 X 2 = 20)**

1. Find the n^{th} derivative of $y = \sin ax$.
2. Find the subnormal of the parabola $y^2 = 4ax$.
3. Is $(0, 0)$ a saddle point for the function $f(x, y) = x^4 + y^4 - 4xy + 1$?
4. Write the steps used in Lagrange's method of undetermined multipliers.
5. Write the $p - r$ equation of a curve.
6. Define evolute of a curve.
7. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, Find the value of $\sum \alpha^2$.
8. Form the equation one of whose roots is $\sqrt{5} - 1$.
9. How many real roots are there in the equation $x^5 - 6x^2 - 4x + 5 = 0$.
10. Is there a root between 1 and 2 for the equation $x^3 + 24x - 50 = 0$?

PART – B

Answer any FIVE questions: **(5 X8 = 40)**

11. If $y = sin^{-1}x$, prove that $(1 - x^2)y_2 - xy_1 = 0$ and

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

12. Find the angle of intersection of the cardioids $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$.
13. Find the minimum value of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
14. Find the radius of curvature at point $\left(\frac{a}{4}, \frac{a}{4}\right)$ to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
15. Find the asymptotes of the curve $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 7 = 0$.
16. Solve the equation $x^3 - 19x^2 + 114x - 216 = 0$, given that the roots are in GP.
17. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

18. Transform the equation $x^3 - 9x^2 + 108 = 0$ into one without the x^2 term.

PART– C

Answer Any TWO Questions: **(2 X 20 = 40)**

19. a) If $y = \left(x + \sqrt{1+x^2}\right)^m$, Prove that $\left(1+x^2\right)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

b) Using Lagrange's multipliers method find the minimum value of u if

$u = a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ **(10 + 10)**

20. a) Find the radius of curvature of the cardioid $r = a(1 - \cos\theta)$

b) Obtain the evolute of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$

(8 + 12)

21. a). Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that $1 - \sqrt{5}$ is a root.

b). Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$ **(8 + 12)**

22. a) Using Cardon's method, solve the equation $x^3 - 6x - 9 = 0$.

b) Using Horner's method find the real root of $x^3 + 6x - 2 = 0$, correct to two places of decimals.

(8 + 12)

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